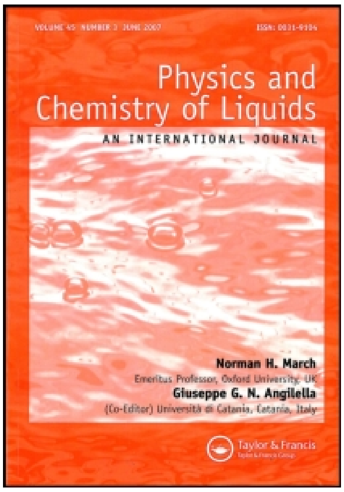

Physics and Chemistry of Liquids
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713646857

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To cite this Article Angilella, G. G. N. , March, N. H. and Pucci, R.(2006) 'Scaling of the superconducting transition temperature in underdoped high-T_c cuprates with a pseudogap energy: Does this support the anyon model of their superfluidity?', Physics and Chemistry of Liquids, 44: 2, 203 — 207

To link to this Article: DOI: 10.1080/00319100600568673

URL: http://dx.doi.org/10.1080/00319100600568673

Physics and Chemistry of Liquids
Vol. 44, No. 2, April 2006, 203–207

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Letter

Scaling of the superconducting transition temperature in underdoped high-T_c cuprates with a pseudogap energy: Does this support the anyon model of their superfluidity?

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(*Received 12 October 2005*)

In earlier work, we have been concerned with the scaling properties of some classes of superconductors, specifically with heavy Fermion materials and with five bcc transition metals of BCS character. Both of these classes of superconductors were three-dimensional but here we are concerned solely with quasi-two-dimensional high-T_c cuprates in the underdoped region of their phase diagram. A characteristic feature of this part of the phase diagram is the existence of a pseudogap (pg). We therefore build our approach around the assumption that $k_B T_c/E_{pg}$ is the basic dimensionless ratio on which to focus, where the energy E_{pg} introduced above is a measure of the pseudogap. Since anyon fractional statistics apply to two-dimensional assemblies, we expect the fractional statistics parameter allowing 'interpolation' between Fermi-Dirac and Bose-Einstein statistical distribution functions as limiting cases to play a significant role in determining $k_B T_c/E_{pg}$ and experimental data are analyzed with this in mind.

Keywords: Superconductivity; Underdoped cuprates; Transition temperature; Anyons

In recent work [1–5], we have been concerned with the connection between superconducting transition temperature T_c and other physical properties in (a) heavy Fermion materials [1,2,5] and (b) five body-centred cubic (bcc) transition metals [4]. For case (a), $k_B T_c$ was shown to correlate strongly with a 'kinetic energy of localization' $\hbar^2/m^*\xi^2$, where m^* is the effective mass of the charge carriers, while ξ is the coherence length. In contrast, in case (b) an intimate relationship between T_c and elastic constants was displayed, showing very directly thereby that these bcc transition metals were BCS-like superconductors.

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Physics and Chemistry of Liquids
ISSN 0031-9104 print: ISSN 1029-0451 online
DOI: 10.1080/00319100600568673

Here, we restrict ourselves solely to the quasi two-dimensional high-T_c cuprates. Because it is known that these materials in the underdoped region of the phase diagram are associated with a pseudogap, it seemed to us natural to consider the ratio T_c/T^*, where $k_B T^*$ is a measure of the energy of the pseudogap. Thus, bearing in mind that anyon fractional statistics [6] are associated with two dimensions, we propose that

$$\frac{T_c}{T^*} = F(T^*, \alpha), \tag{1}$$

where the, as yet unknown, function F depends on T^* itself, and on the anyon fractional statistics parameter α chosen to lie in the range between 0 and 1. In addition, we must expect that, whereas in BCS theory $T^* \to \Theta_D$, where Θ_D is the Debye temperature, and no anyons exist in three dimensions, there will be some coupling parameter as for the strength of the electron-phonon interaction in BCS theory. While we anticipate that equation (1) should obtain in the quasi-2D high-T_c cuprates independent of any particular simplifying model, we intend below to exemplify our assumptions embodied in equation (1) by appealing to a very recent and specific 2D model [7,8], which we propose to generalize by heuristic arguments to embrace the fractional statistics parameter α.

The mean-field T_c has been correlated to the quantity E_{pg} related to a measure of the pseudogap by Fine [7] within a model [8] based on the existence of stripes in the CuO_2 planes of high-T_c superconductors. The resulting correlation can be presented as [7]

$$k_B T_c = \frac{g^2}{8E_{pg}} \frac{e^{E_{pg}/k_B T_c} - 1}{e^{E_{pg}/k_B T_c} + 1}, \tag{2}$$

where g is the coupling constant between stripes and the antiferromagnetic (AFM) domains between the stripes.

We now turn to the heuristic generalization of Fine's model for T_c displayed in equation (2). We appeal first to the simple collision model used by one of us [9] (see also Ref. [10]), which allowed a partial unification of Fermi-Dirac (FD), Bose-Einstein (BE), and anyon fractional statistics. If the statistical distribution function in each case is denoted by $f(\epsilon)$, with ϵ the particle energy, then the unifying equation in Refs. [9,10] was

$$\frac{1}{f(\epsilon)} = \exp\left(\frac{\epsilon - \mu}{k_B T}\right) + a, \tag{3}$$

where a was assumed to depend only on α. Then the choice $a = 2\alpha - 1$ gave a correctly for BE statistics with $\alpha = 0$ and $a = -1$, and for FD statistics with $\alpha = 1$ and $a = 1$. The subsequent microscopic theory of Wu [11] showed that a away from these endpoints $\alpha = 0$ and 1 also depended on $\epsilon/k_B T$. Wu's result takes the form

$$n(\epsilon - \mu) = \frac{1}{w[e^{(\epsilon-\mu)/k_B T}] + \alpha}, \tag{4}$$

where ϵ is the energy level, μ the chemical potential, and α is the fractional statistics parameter introduced above, ranging between the limiting values $\alpha = 0$ and $\alpha = 1$, for which equation (4) reduces to the familiar Bose-Einstein and Fermi-Dirac distributions, respectively. The case $\alpha = 1/2$ refers to 'semions'. In equation (4), the 'generalized exponential' $w(\zeta)$ obeys the functional equation [11]

$$w^{\alpha}(\zeta)[1 + w(\zeta)]^{1-\alpha} = \zeta \equiv e^{(\epsilon-\mu)/k_{\mathrm{B}}T}. \tag{5}$$

We then propose to generalize Fine's equation (2) above to embrace the cases of anyon statistics as follows:

$$\frac{k_{\mathrm{B}}T_{\mathrm{c}}}{E_{\mathrm{pg}}} = \frac{g^2}{8E_{\mathrm{pg}}^2}\frac{w(e^{E_{\mathrm{pg}}/k_{\mathrm{B}}T_{\mathrm{c}}}) + \alpha}{2e^{E_{\mathrm{pg}}/k_{\mathrm{B}}T_{\mathrm{c}}} - w(e^{E_{\mathrm{pg}}/k_{\mathrm{B}}T_{\mathrm{c}}}) - \alpha}, \tag{6}$$

where we have divided both sides of equation (2) by E_{pg}, in order to form the dimensionless ratios $k_{\mathrm{B}}T_{\mathrm{c}}/E_{\mathrm{pg}}$ and $8E_{\mathrm{pg}}^2/g^2$. It may be checked that equation (6) reduces to Fine's formula, equation (2), in the limit $\alpha = 0$.

Figure 1 shows the numerical solution of equation (6) for the inverse ratio $E_{\mathrm{pg}}/k_{\mathrm{B}}T_{\mathrm{c}}$ plotted as a function of the dimensionless variable $8E_{\mathrm{pg}}^2/g^2$, with g denoting, as mentioned above, Fine's coupling constant. The different curves are characterized by the specific values of the parameter α recorded in figure 1. Above the value $E_{\mathrm{pg}}/k_{\mathrm{B}}T_{\mathrm{c}} \simeq 1$, these curves are (i) rather linear and (ii) have slopes which vary only weakly with α until it reaches around the semion value $\alpha = 1/2$. We have also plotted, for completeness of the consequences of equation (6), the curve for $\alpha = 1$ though we do not anticipate it will have significance for the high-T_{c} cuprates, as the ensuing discussion will indicate.

This is the point at which we invoke explicit experimental data for the high-T_{c} cuprates. The review of Timusk and Statt [12] compares T_{c} with T^* determined

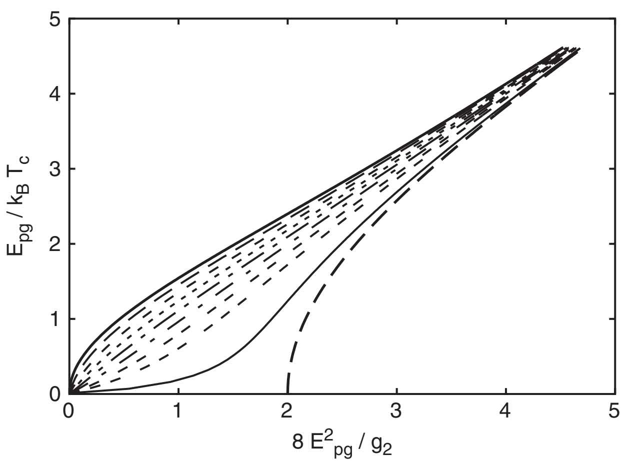

Figure 1. Showing our proposed generalization of Fine's formula, equation (6), to the case of anyon statistics. The anyon parameter ranges from $\alpha = 0$ (top curve, solid line) to $\alpha = 1$ (bottom curve, dashed line).

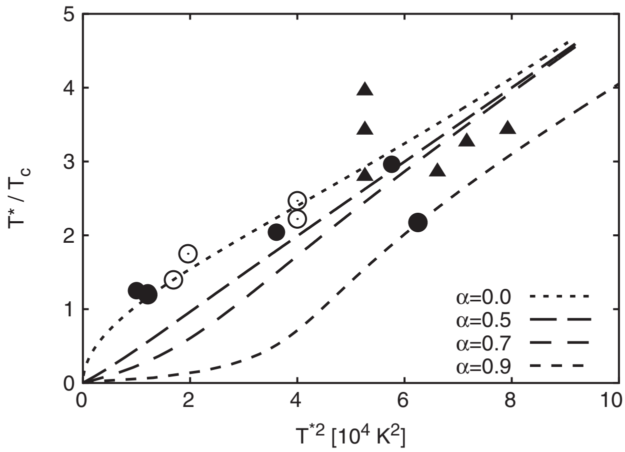

Figure 2. Ratio of pseudogap and critical temperature, T^*/T_c, versus square of pseudogap temperature, T^{*2}, for several high-T_c superconductors in the pseudogap regime (data taken from Tables 1 and 2 of Ref. [12]). Filled and open circles refer to pseudogap temperatures T^* from Knight shift and NMR relaxation rates measurements, respectively, while filled triangles refer to maximum pseudogap, as derived from in-plane optical scattering measurements [12]. Lines are instances of equation (6), with $k_\mathrm{B}T^* \equiv E_\mathrm{pg}$, and $g^2/8 = 2 \cdot 10^4\,\mathrm{K}^2$ for the cases $\alpha = 0, 0.5, 0.7$ cases, and $g^2/8 = 2.4 \cdot 10^4\,\mathrm{K}^2$ for the case $\alpha = 0.9$.

by Knight shift and by NMR relaxation experiments in their Table 1, as well as by in-plane optical scattering measurements in their Table 2. In figure 2, and in the absence of knowledge of what to take for the coupling constant g, we assume as a starting point that it is material independent and plot therefore, but now from experiment, T^*/T_c versus T^{*2} (cf. figure 1). There is already clear evidence for the shape displayed in figure 1.

For most of the points in figure 2, α small is the appropriate choice. But for the highest T_c material $HgBa_2Ca_2Cu_3O_{8+\delta}$ with $T_\mathrm{c} = 115\,\mathrm{K}$ and $T^* = 250\,\mathrm{K}$ [12], we can fit this rather isolated point by taking $\alpha = 0.9$ and making a change in coupling strength. It is tempting therefore to believe that the highest T_c cuprates with $T_\mathrm{c} \sim 140\,\mathrm{K}$ might correspond also to a large fractional statistics parameter α.

To summarize, we have been able to give some substantial support to our basic assumption equation (1) by making direct use of experimental data on the pseudogap. By heuristic generalization of Fine's mean field solution of his 2D stripe model, we have exhibited how we expect the fractional statistics parameter α to influence the behaviour of T^*/T_c. Present indications are that higher T_c materials should correspond to larger α, but the material dependence of the basic coupling strength (g, in Fine's model) remains to be studied more fundamentally.

Acknowledgements

One of us (NHM) acknowledges that his contribution to this Letter was made during a stay in the Department of Physics and Astronomy, University of Catania. He thanks Professors F. Catara, R. Pucci, and E. Rimini for generous hospitality.

NHM wishes also to acknowledge travel and maintenance support from the University of Antwerp, and to thank especially Professor D. Van Dyck for his continuing hospitality.

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